SCOPE AND RESOLUTION IN LOCAL SEARCH OPTIMISATION

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Abstract

Local search methods abound for approximating the optimal solution to large-scale problems in combinatorial optimization (CO). Tabu searches, simulated annealing techniques and evolutionary/genetic/ant colony algorithms all seek to generate a *solution sequence* in a solution space from an initial to final local minimum. Each exploits the characteristics of the search neighbourhood in its own way but all share the common objective of attempting to maximize the range of the search whilst preserving its validity or usefulness. In the case of NP-hard or NP-complete problems, all are (presumably) eventually defeated by the (generally assumed) inherent complexity of the problems themselves.

This paper takes a slightly oblique look at the local search process. It uses, as a basis for investigation, the Asymmetric Traveling Salesman Problem (ATSP), one of the most general and representative of all CO problems, and a specially-developed software package (TSPGen - see below) to analyse the behaviour of the commonest techniques. Starting from a nearest-neighbour initial solution, combinations of *2-OPT*, *3-OPT*, *4-OPT* a configurable *k-OPT* and the Lin-Kernigham searches are applied manually, automatically or at random. Using TSPGen, the essential intractability of the problem can be observed at once.

The *scope* of a local search iteration may be defined in terms of its search neighbourhood. (For example, *3-OPT* has larger scope than *2-OPT*.) A process with large scope may be better able to find a lower local minimum beyond intermediate larger ones. However, dependent upon the nature of the search, any given algorithm may not be guaranteed to find the lowest local minimum if its *resolution*, its ability to discern solutions within its scope, is crude. (Again, as an example, a *3-OPT* search may overlook *2-OPT* gains.) More significantly, no greedy approach to generating a solution sequence can guarantee optimality, nor can a single search process in isolation; either the scope or resolution will be inadequate. Using the notion of a *search sequence*, generating the solution sequence, TSPGen finds ATSP instances in which a mixed search sequence produces superior solutions to a fixed one. (Assuming a variable search to include each fixed search, it can never perform worse.)

Recognising that an optimal or improved solution sequence may result from a combination of iterative steps from different search patterns suggests naturally a parameter-based decision tree approach to local search. At the highest resolution/smallest scope, *2-OPT*, *3-OPT*, …, etc. may be broken down into separate search patterns *2(j)-OPT*, *3(j)-OPT*, …, etc, with the new parameter, j, representing the distance between nodes on exchanged edges. At a larger scope/smaller resolution, the parameter, k, in the *k-OPT* provides the necessary variation. *k(j)-OPT* in its general form considers search patterns drawn from *2(1)-OPT*, *2(2)-OPT*, …, *3(1)-OPT*, *3(2)-OPT*, …, etc. If J and K denote the range of j and k respectively then from a full search pattern (J^*, K^*), an optimal solution may be guaranteed.

An optimal tree-search of this form is (unavoidably unless $P = NP$) of exponential time complexity. However, the value of this parameter-based approach is that J and K can be chosen to suit the available computing power. A single generic algorithm may thus be used in a range of applications and experimental comparisons made fairly and appropriately. The implementation of the algorithm in this form is straightforward and it performs well.

Software

The TSPGen Software, developed at University of Wales, NEWI Wrexham, allows small and larger symmetric and asymmetric TSP instances to be generated randomly and analysed graphically and numerically. Its essential feature is to permit local search iterations to proceed both automatically and on a stepwise basis, allowing a variety of search and solution sequences to be constructed quickly or examined in more detail. It also offers a number of additional analytic features used in this paper.